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Effect of bottom topography on the unsteady behaviour of an elastic plate floating on shallow water☆

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ABSTRACT

The unsteady behaviour of a thin elastic plate in the form of a strip of constant width and infinite length, floating on the surface of an ideal and incompressible liquid, is investigated within the limits of the linear shallow-water theory. The unsteady behaviour of the plate is caused by initial disturbances or an external load. The depth of the liquid under the plate is variable. It is assumed that all the flow characteristics are independent of the coordinate along the plate. The deflection of the plate is sought in the form of an expansion in eigenfunctions of the oscillations in a vacuum with time-varying amplitudes. The problem reduces to solving an infinite system of ordinary differential equations for the unknown amplitudes. The behaviour of the plate is investigated for different actions and shapes of bottom irregularities. It is shown that the bottom topography can have a considerable effect on the deformation of the plate.

The hydroelastic behaviour of thin plates floating on the surface of a liquid is of interest in a number of practical applications: floating platforms, ice fields and breakwaters. The large number of investigations into the behaviour of floating elastic plates under the action of incoming surface waves and an external load (see the Refs. 1–3, for example) has mainly been carried out on the assumption that the liquid depth is constant. In reality, however, the irregularity of the bottom and the corresponding changes in the liquid depth can have an appreciable effect on the hydroelastic behaviour of the plate when a floating structure is located close to the shore. The effect of the bottom topography has recently been investigated in the linear problem of the scattering of periodic surface waves by a floating elastic Plate[4–6] on the assumption that the liquid flow and the plate deformation are periodic functions of time. The two-dimensional problem has been considered[4] for a liquid of finite depth. The boundary elements method was used in the numerical solution of Laplace's equation in a finite domain which encompasses the irregularity and the plate. The behaviour of a rectangular plate has been studied numerically using the method of finite differences[5] in the case of shallow water and using the finite elements method[6] in the case of a finite liquid depth. A comparison, carried out in Ref. 5, of the numerical and experimental results for the oscillation amplitudes of a plate showed good agreement. It has been noted[6] that the effect of an irregular bottom is particularly noticeable in the case of a shallow depth and long incoming waves.

A method of solving the linear time-varying problem of a floating elastic beam plate is proposed in this paper. The following cases are considered as examples: the incidence of a localized surface wave on a plate, the initial deformation of a plate, and the action of a moving external load on it. The first and second cases have been studied earlier in the case of a regular bottom[7], as well as the third case[8]. The solution of the problem in the first case will initially be described in detail, and the changes which are introduced in the other cases are then briefly indicated.

1. Formulation of the problem

Suppose a homogeneous elastic beam of length $2L$ floats freely on the surface of an ideal incompressible liquid. The liquid surface, which is not covered by the beam, is free. The domain S, occupied by the liquid, is subdivided into three parts: $S_1(|x| < L)$, $S_2(x < -L)$ and S_3 $(x > L)$, where x is the horizontal coordinate. When there is no plate, the depth of the liquid is equal to $H(x)$ in the domain S_1. In the semi-infinite domains S_2 and S_3, the depths of the liquid are assumed to be constant and equal to H_1 and H_2 respectively. For simplicity,

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the change in the liquid depth is assumed to be continuous, that is, $H(-L) = H_1$ and $H(L) = H_2$ but an abrupt change in the depth when $|x| = L$ can also be considered. When the plate is present, the liquid depth in the domain S_1 is reduced by the value of the settling of the plate d and is equal to $h(x) - H(x) - d$. It is assumed that the maximum liquid depth is small compared with the length of the surface waves and flexural-gravitational waves, and the shallow water approximation is used. The velocity potentials describing the liquid motion in the domains S_j, are equal to $\phi_j\ (x, t)$ $(j = 1, 2, 3)$, where t is the time.

We will assume that a localized surface wave, in which the vertical displacement of the liquid is equal to $\eta_0(x, t) = f(x - \sqrt{gH_1}t)$, runs onto the beam from the left. The function $f(\xi)$ is only non-zero when $|\xi| < c$. Such a wave can arise as the result of the collapse of the elevation of the free surface at an instant $t = t_0$ subject to the condition that all the liquid is initially at rest. It is well known[9] that, when $t > t_0$, the free surface represents two isolated waves moving in opposite directions without deformation and with a velocity $\sqrt{gH_1}$. The amplitudes of these waves are equal to half the amplitude of the initial elevation and the width of the domain occupied by each of these waves is equal to the width of the domain of the initial elevation. It is assumed that, at the instant $t = 0$, the plate and the liquid are at rest in the domains S_1 and S_3 and the localized disturbance reaches the left edge of the plate in the domain S_2. When $t > 0$, oscillations of the plate and the liquid commence in the domain S_1 which cause wave disturbances diverging from the plate in the domains S_2 and S_3.

The normal bending of an Euler beam $w(x, t)$ is described by the equation

$$D\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + g\rho w + \rho\frac{\partial \phi_1}{\partial t} = 0, \quad x \in S_1 \tag{1.1}$$

where D is the cylindrical stiffness of the plate, m is its specific mass, ρ is the liquid density and g is the acceleration due to gravity.

According to linear shallow-water theory, the relation

$$\frac{\partial w}{\partial t} = -\frac{\partial}{\partial x}\left(h(x)\frac{\partial \phi_1}{\partial x}\right), \quad x \in S_1 \tag{1.2}$$

holds. In domains outside the plate, the velocity potentials satisfy the equations

$$\frac{\partial^2 \phi_2}{\partial t^2} = gH_1\frac{\partial^2 \phi_2}{\partial x^2}, \quad x \in S_2, \quad \frac{\partial^2 \phi_3}{\partial t^2} = gH_2\frac{\partial^2 \phi_3}{\partial x^2}, \quad x \in S_3 \tag{1.3}$$

The elevations of the free surface $\eta_2(x, t)$ and $\eta_3(x, t)$ in the domains S_2 and S_3 respectively are determined from the relations

$$\eta_j = -\frac{1}{g}\frac{\partial \phi_j}{\partial t}, \quad x \in S_j, \quad j = 2, 3$$

Conditions for a free edge, that is, the bending moment and the shearing force are equal to zero

$$|x| = L\colon \frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0$$

are set up on the edge of the beam, and the pressure and mass continuity conditions

$$x = -L\colon \frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_2}{\partial t}, \quad \frac{\partial \phi_1}{\partial x} = \frac{H_1}{h_1}\frac{\partial \phi_2}{\partial x}; \quad x = L\colon \frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_3}{\partial t}, \quad \frac{\partial \phi_1}{\partial x} = \frac{H_2}{h_2}\frac{\partial \phi_3}{\partial x} \tag{1.4}$$

where

$$h_{1,2} = H_{1,2} - d, \quad d = m/\rho$$

must also be satisfied. Far from the beam

$$x \to -\infty\colon \partial\phi_2/\partial x \to 0; \quad x \to \infty\colon \partial\phi_3/\partial x \to 0$$

The initial conditions have the form

$$t = 0\colon w = \eta_3 = \frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_3}{\partial t} = 0, \quad \eta_2 = \eta_0, \quad \frac{\partial \phi_2}{\partial t} = -g\eta_0 \tag{1.5}$$

We will now change to dimensionless variables, taking L as the unit of length and $\sqrt{L/g}$ as the unit of time. The following dimensionless coefficients are used

$$\delta = \frac{D}{\rho g L^4}, \quad \gamma = \frac{d}{L}$$

2. The method of normal modes

We will seek the beam deflection in the form of an expansion in the eigenfunctions of the oscillation of a beam with free ends in a vacuum:

$$w(x,t) = \sum_{n=0}^{\infty} X_n(t) W_n(x) \tag{2.1}$$

The functions $X_n(t)$ are to be determined and the functions $W_n(x)$ are the solutions of the following eigenvalue problem in dimensionless variables

$$W_n^{IV} = \lambda_n^4 W_n, \quad |x| \le 1$$

$$|x| = 0:\ W'_{2n} = W_{2n+1} = 0;\ |x| = 0: \quad W''_n = W'''_n = 0$$

A prime denotes differentiation with respect to x.

These solutions have the form

$$W_0 = 1/\sqrt{2}, \quad W_1 = \sqrt{3/2}x, \quad W_{2n} = D_{2n}[\cos(\lambda_{2n}x) + S_{2n}\operatorname{ch}(\lambda_{2n}x)]$$

$$W_{2n+1} = D_{2n+1}[\sin(\lambda_{2n+1}x) + S_{2n+1}\operatorname{sh}(\lambda_{2n+1}x)]$$

where

$$S_n = \cos\lambda_n/\operatorname{ch}\lambda_n, \quad D_n = 1/\sqrt{1+(-1)^n S_n^2}$$

The eigenvalues λ_n are determined from the equation

$$\operatorname{tg}\lambda_n + (-1)^n \operatorname{th}\lambda_n = 0\ (n \ge 2), \quad \lambda_0 = \lambda_1 = 0$$

The functions $W_n(x)$ form a complete orthogonal system normalized in the following manner

$$\int_{-1}^{1} W_n(x) W_m(x) dx = \delta_{nm}$$

where δ_{nm} is the Kronecker delta.

Substituting expansion (2.1) and initial conditions (1.5) into Eq. (1.1), multiplying the resulting relations by $W_m(x)$ and integrating them with respect to x from −1 to 1, we obtain the following system of ordinary differential equations with the initial conditions

$$\gamma\ddot{X}_m + (\delta\lambda_m^4 + 1)X_m + F_m(t) = 0, \quad X_m(0) = \dot{X}_m(0) = 0 \tag{2.2}$$

Here,

$$F_m(t) = \int_{-1}^{1} W_m \frac{\partial\phi_1}{\partial t} dx \tag{2.3}$$

where the dot denotes differentiation with respect to time.

We will seek the solution for $\phi_1(x,t)$ in the form

$$\phi_1(x,t) = \sum_{n=0}^{\infty} \dot{X}_n(t)\Psi_n(x) + q(x,t)$$

where the functions $\Psi_n(x)$ satisfy the equation

$$\Psi'_n(x) = -V_n(x)/h(x), \quad V'_n(x) = W_n(x)$$

and have the form

$$\Psi_n(x) = -\int_{-1}^{x} \frac{V_n(\xi)}{h(\xi)} d\xi; \quad V_0 = \frac{x}{\sqrt{2}}, \quad V_1 = \frac{1}{2}\sqrt{\frac{3}{2}}x^2$$

$$V_{2n} = \frac{D_{2n}}{\lambda_{2n}}[\sin(\lambda_{2n}x) + S_{2n}\operatorname{sh}(\lambda_{2n}x)], \quad V_{2n+1} = \frac{D_{2n+1}}{\lambda_{2n+1}}[S_{2n+1}\operatorname{ch}(\lambda_{2n+1}x) - \cos(\lambda_{2n+1}x)]$$

The function $q(x, t)$ is unknown and, according to Eq. (1.2) and initial conditions (1.5), has the form

$$q(x,t) = Q(x)u(t) + \upsilon(t), \quad Q(x) = \int_{-1}^{x} \frac{d\xi}{h(\xi)}, \quad u(0) = \upsilon(0) = 0$$

The functions $u(t)$ and $\upsilon(t)$ are determined from the matching conditions (1.4).

We next consider the behaviour of the solution in the domains S_2 and S_3 and seek the solution for $\phi_2(x, t)$ in the form

$$\phi_2(x,t) = \phi_0(x,t) + \psi(x,t)$$

where $\phi_0(x, t)$ is the potential of the incoming wave, which is determined from the relation

$$\partial\phi_0/\partial x = \eta_0/\sqrt{H_1}$$

The function $\psi(x, t)$ describes the velocity potential of the reflected wave. According to Eq. (1.3), the solution for $\psi(x, t)$ can be sought in the form

$$\psi(x,t) = \begin{cases} A((x+1)/\sqrt{H_1}+t), & -(1+\sqrt{H_1}t) < x < -1 \\ 0, & x < -(1+\sqrt{H_1}t) \end{cases} \tag{2.4}$$

A similar representation also holds for the function $\phi_3(x, t)$ which describes the velocity potential of the transmitted wave

$$\phi_3(x,t) = \begin{cases} B(t-(x-1)/\sqrt{H_2}), & 1 < x < 1+\sqrt{H_2}t \\ 0, & x > 1+\sqrt{H_2}t \end{cases} \tag{2.5}$$

The functions $A(\xi)$ and $B(\xi)$ are unknown. Using the matching conditions (1.4), we obtain the following differential equations for these functions

$$\dot{A} = (u + \dot{X}_0R_0 - \dot{X}_1R_1)/\sqrt{H_1} - \alpha(t), \quad \dot{B} = (\dot{X}_0R_0 + \dot{X}_1R_1 - u)/\sqrt{H_2} \tag{2.6}$$

with the initial conditions

$$A(0) = B(0) = 0$$

Here,

$$\alpha(t) = \eta_0(-1,t), \quad R_n = V_n(1), \quad R_0 = \frac{1}{\sqrt{2}}, \quad R_1 = \frac{1}{2}\sqrt{\frac{3}{2}}, \quad R_n = 0 \text{ when } n \geq 2$$

Using the relations presented above, we obtain an expression for the function $F_m(t)$ (2.3) in the form

$$F_m(t) = \sum_{n=0}^{\infty} \ddot{X}_n(t)(\Lambda_n R_m + C_{nm}) + (\beta R_m + \Lambda_m)\dot{u} + \\ + \sqrt{2}\delta_{m1}[(u + \dot{X}_0R_0 - \dot{X}_1R_1)/\sqrt{H_1} - 2\alpha(t)]$$

where

$$C_{nm} = \int_{-1}^{1} \frac{V_n(x)V_m(x)}{h(x)}dx, \quad \Lambda_n = \Psi_n(1), \quad \beta = Q(1)$$

The final system of differential equations for determining the oscillations of the beam has the form

$$\sum_{n=0}^{\infty} \ddot{X}_n(\gamma\delta_{nm} + \Lambda_n R_m + C_{nm}) + \sqrt{2}\delta_{m0}\left[\frac{1}{\sqrt{H_1}}(u + \dot{X}_0R_0 - \dot{X}_1R_1) - 2\alpha(t)\right] + \\ + (\delta\lambda_m^4 + 1)X_m + (\beta R_m + \Lambda_m)\dot{u} = 0$$

$$\dot{u} = \frac{1}{\beta}\left[\left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right)\dot{X}_0R_0 + \left(\frac{1}{\sqrt{H_1}} + \frac{1}{\sqrt{H_2}}\right)(\dot{X}_1R_1 - u) - \sum_{n=0}^{\infty}\ddot{X}_n\Lambda_n + 2\alpha(t)\right] \tag{2.7}$$

After the functions $X_n(t)$ and $u(t)$ have been determined, it is possible to find all the characteristics of the motion of the liquid and the elastic beam. For example, for the vertical elevations of the free surface we obtain

$$\eta_2(x,t) = \eta_0(x,t) + \zeta(x,t) \text{ in domain } S_2$$

$$\zeta(x,t) = \begin{cases} -\dot{A}((x+1)/\sqrt{H_1}+t), & -(1+\sqrt{H_1}t) < x < -1 \\ 0, & x < -(1+\sqrt{H_1}t) \end{cases}$$

$$\eta_3(x,t) = \begin{cases} -\dot{B}(t-(x-1)/\sqrt{H_2}), & 1 < x < 1+\sqrt{H_2}t \\ 0, & x > 1+\sqrt{H_2}t \end{cases} \text{ in domain } S_3$$

The functions $\dot{A}(\xi)$ and $\dot{B}(\xi)$ are determined from Eqs. (2.6).

3. The energy relation

We will now determine the change with time of the energy of the transmitted and reflected waves. The total energy of the incoming wave is equal to[9]

$$E_0 = \int_{-(1+2c)}^{-1} \eta_0^2(x,0)dx$$

This energy is transferred to the oscillations of the elastic plate and to the scattered (transmitted or reflected) surface waves. When $t \to \infty$, the oscillations of the plate attenuate and it is restored to its initial horizontal position. The energy of the reflected wave is equal to

$$E_r(t) = \int_{-(1+\sqrt{H_1}t)}^{-1} \zeta^2(x,t)dx = \sqrt{H_1}\int_0^t \dot{A}^2(\xi)dx\xi$$

and the energy of the transmitted wave is equal to

$$E_t(t) = \int_1^{1+\sqrt{H_2}t} \eta_3^2(x,t)dx = \sqrt{H_2}\int_0^t \dot{B}^2(\xi)d\xi$$

There is no energy dissipation in this problem and, consequently,

$$\lim_{t\to\infty}[E_r(t)+E_t(t)] = E_0$$

Satisfaction of this equality can serve as a criterion of the accuracy of the method of solution used.

4. The action of a load on the beam

It is assumed that the deformation of the beam is caused by its initial disturbances and an external load and that the liquid outside the beam is initially at rest. In this case, Eq. (1.1) takes the form

$$D\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + g\rho w + \rho\frac{\partial \phi_1}{\partial t} = -p_e(x,t) \ (x \in S_1) \tag{4.1}$$

The specified function $p_e(x, t)$ describes the external load on the beam. The initial conditions

$$\begin{aligned} &\phi_1(x,0) = \phi_1^0(x), \quad w(x,0) = w_0(x), \quad \partial w/\partial t|_{t=0} = w_1(x) \\ &\phi_2(x,0) = \phi_3(x,0) = \eta_2(x,0) = \eta_3(x,0) = 0 \end{aligned} \tag{4.2}$$

contain functions which must satisfy relations (1.2) and (1.4).

Using expansion (2.1), instead of (2.2), we obtain the following system of differential equations with the initial conditions

$$\gamma\ddot{X}_m + (\delta\lambda_m^4+1)X_m + F_m(t) = -Z_m(t)$$

$$X_m(0) = \int_{-1}^{1} w_0(x)W_m(x)dx, \quad \dot{X}_m(0) = \int_{-1}^{1} w_1(x)W_m(x)dx$$

Here,

$$Z_m(t) = \int_{-1}^{1} p_e(x,t) W_m(x) dx$$

The representations for the solutions in the domains S_2 and S_3 are identical to representations (2.4) and (2.5) respectively, with the sole difference that the function $A(\xi)$ is determined from the first equation of (2.6) when $\alpha(t) \equiv 0$.

The final system of differential equations now has the form of system (2.7), if we put $\alpha(t) \equiv 0$ in it and the zero right-hand side of the first equation is replaced by the function $-Z_m(t)$.

5. Numerical results

Using the reduction method, we replace the infinite series in the expansion (2.1) with a sum of N terms. The system of ordinary differential equations (2.7), and the system analogous to it, which has just been described, were solved numerically using a fourth-order Runge–Kutta

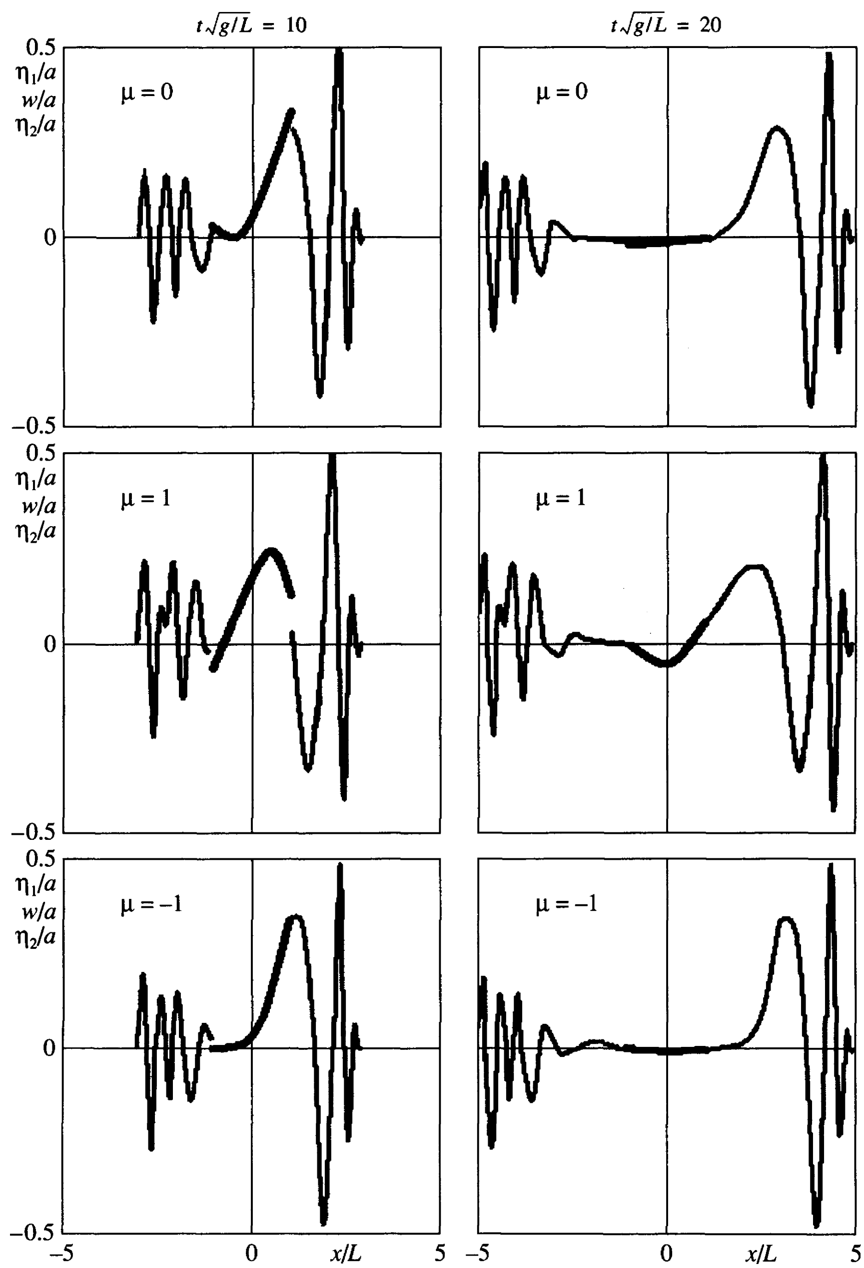

Fig. 1.

method. The following initial parameters were used

$$L = 500 \text{ m}, \quad d = 5 \text{ m}, \quad \delta = 5 \cdot 10^{-3}, \quad \rho = 10^3 \text{kg/m}^3, \quad H_1 = 20 \text{ m}, \quad H_2 = 10 \text{m}$$

In all the calculations, the number of the beam oscillation modes $N = 20$ and a further increase in N hardly changes the result.

Three forms of bottom topography are considered: a hill, a hollow and a uniform decline. The relief of the bottom, which is described by the relation

$$H(x) = H_1\left\{1 + \frac{\mu}{2}\left[\left(\frac{x}{L}\right)^2 - 1\right]\right\} \tag{5.1}$$

simulates a hill when $\mu > 0$ and a hollow when $\mu < 0$. A uniform decline is described by the relation

$$H(x) = [(H_2 - H_1)x + H_1 + H_2]/2$$

In investigating the incidence of a localized surface wave on an elastic beam, the shape of the beam was chosen in the form

$$f(\xi) = \begin{cases} \frac{a}{2}\left(1 + \cos\frac{\pi\xi}{c}\right), & |\xi| < c \\ 0, \quad |\xi| > c \end{cases}; \quad \xi = x - t\sqrt{gH_1} - x_0$$

The total energy of such a wave is constant in time and equal to

$$E_0 = \rho g \int_{-c}^{c} f^2(\xi) d\xi = \frac{3}{4}\rho g a^2 c$$

The behaviour of the form of the free surface and of the beam at the instants $t\sqrt{g/L} = 10$ (on the left) and $t\sqrt{g/L} = 20$ (on the right) when $x_0/L = -1.25$, $c/L = 0.25$ is shown in Fig. 1. The upper part of Fig. 1 refers to the case of a uniform bottom with a depth H_1, and the middle and lower parts are obtained for a bottom irregularity of the form (5.1) when $\mu = 1$ (a hill) and $\mu = -1$ (a hollow). The behaviour of η_2/a when $x/L < 1$ is shown, the behaviour of w/a when $|x|/L < 1$ is shown by the solid line and the behaviour of η_3/a. When $t\sqrt{g/L} = 10$, the initial wave passes under the plate, which leads to its deformations. The deflections of the plate as well as the behaviour of the free surface depend very much on the shape of the bottom.

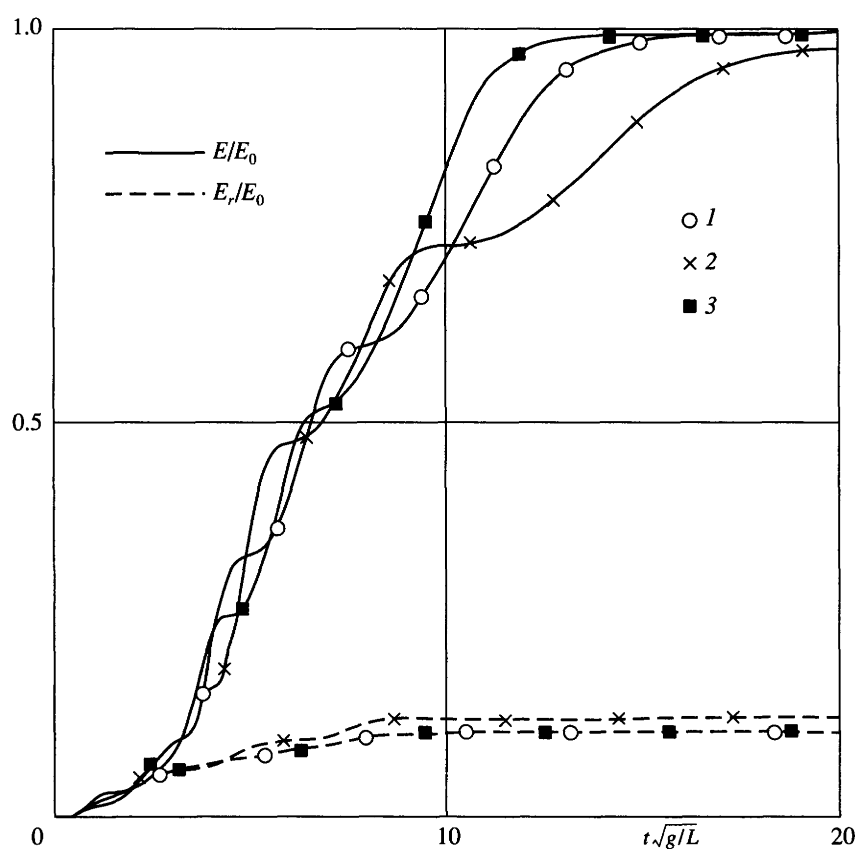

Fig. 2.

When $t\sqrt{g/L} = 20$, a significant portion of the initial wave energy of the incoming wave was transformed into the transmitted wave. However, the form of the transmitted wave is very different from the form of the initial wave. This difference is due both to the properties of the plate and the shape of the bottom. In the case of a uniform bottom and when there is a hollow in the bottom, the oscillations of the plate become very weak by this time but, when there is a hill, they still remain quite appreciable.

The change in the total energy of the reflected wave and the combined energy of the transmitted and reflected waves $E(t) = E_r(t) + E_r(t)$ with time is shown in Fig. 2 for all three types of bottom relief. The results for a uniform bottom, a hill and a hollow are labelled with the numbers 1, 2 and 3 respectively. The results for the combined energy are represented by the solid curves and the results for the reflected wave by the dashed curves. It can be seen that the limiting value of the combined energy is attained most rapidly in the case of a hollow. Consequently, in this case, the oscillations of the plate attenuate earlier than in the other two cases. The plate oscillates for significantly longer when there is a hill on the bottom. In this case, the energy of the reflected waves is somewhat greater, although the difference between the three cases considered is quite insignificant for the reflected waves. Only about 10% of the initial energy is scattered into the reflected waves and the main part of the energy passes through the plate and propagates in the form of transmitted waves.

We next consider the behaviour of a beam due to its initial deformation in a liquid at rest. The following functions, which define the initial conditions (4.2), are used

$$\phi_1^0 = w_1 = 0, \quad w_0(x) = \frac{a}{2}\left[1 + \cos\frac{\pi(x - x_0)}{c}\right]$$

There is no external load and, consequently, $p_e = 0$ in Eq. (4.1). The behaviour of the normal deflections of the beam is shown in Fig. 3 for a function $w_0(x)$ which is symmetrical about the origin of coordinates when $x_0 = 0$, $c = L$ (on the left) and for an asymmetric function $w_0(x)$ when $x_0 = c = L/2$ (on the right) for the two instants: $t\sqrt{g/L} = 2.5$ and $t\sqrt{g/L} = 10$. The results for a uniform bottom, a hill, a hollow

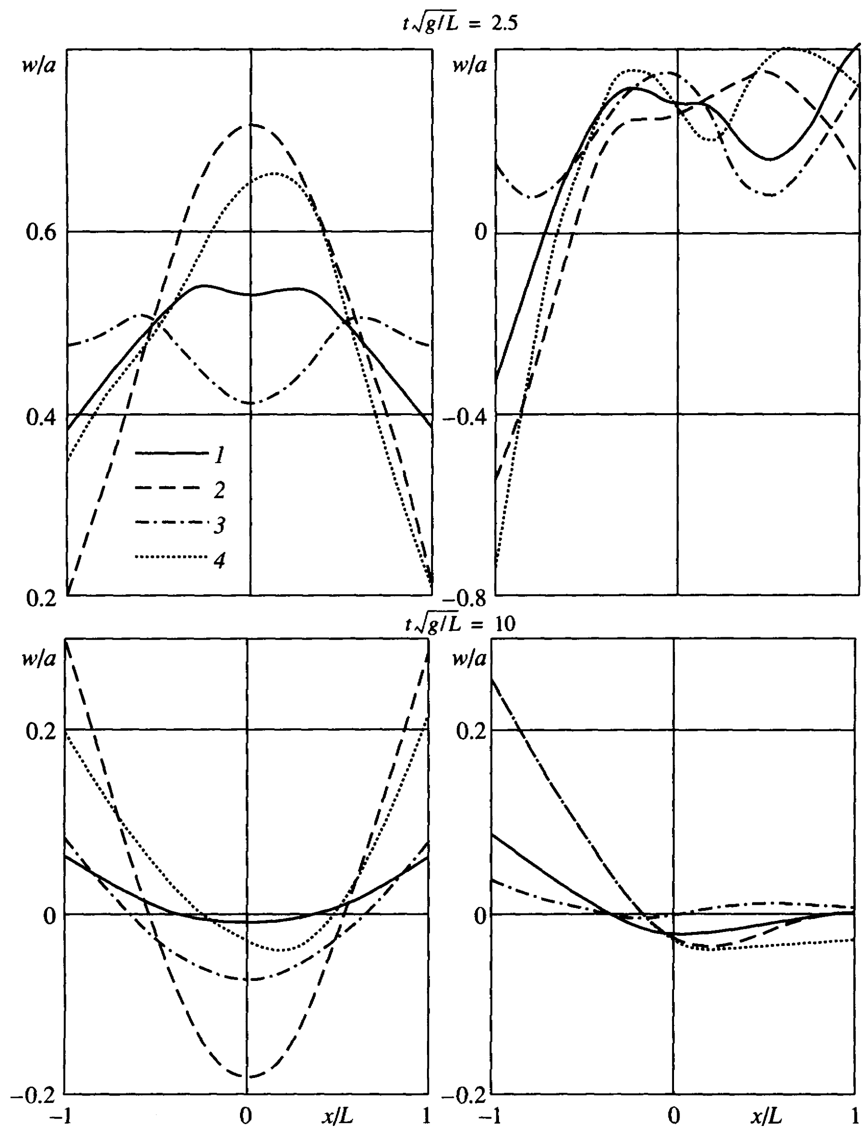

Fig. 3.

and a slope correspond to curves 1–4. Like the results for an initial elevation of the free surface, the oscillations of the beam depend considerably on the shape of the bottom. The most long-lived oscillations are observed when there is a hill on the bottom, particularly in the case of a symmetric initial deformation of the beam. The difference in the initial shape between the symmetric and asymmetric cases leads to the fact that higher modes are excited in the asymmetric case which, however, decay more rapidly with time than the lower modes.

The effect of a moving load is considered using an example which simulates the landing of an aircraft. It is assumed that the beam and the liquid are at rest at the initial instant and that a load with a velocity υ_0 smoothly touches upon the beam in a domain with its centre at the point $x=x_0$ and subsequently moves while uniformly slowing down until it comes completely to rest at the point $x=x_1$ at the instant $t=t_1\equiv 2(x_1-x_0)/\upsilon_0$. The change in the velocity of the load $\upsilon(t)$ with time when $t<t_1$ is equal to $\upsilon(t)=\upsilon_0(1-t/t_1)$ and the position of the epicentre of the load $s(t)$ is given by the relation

$$s(t) = x_0 + \upsilon_0 t\left(1-\frac{t}{2t_1}\right)$$

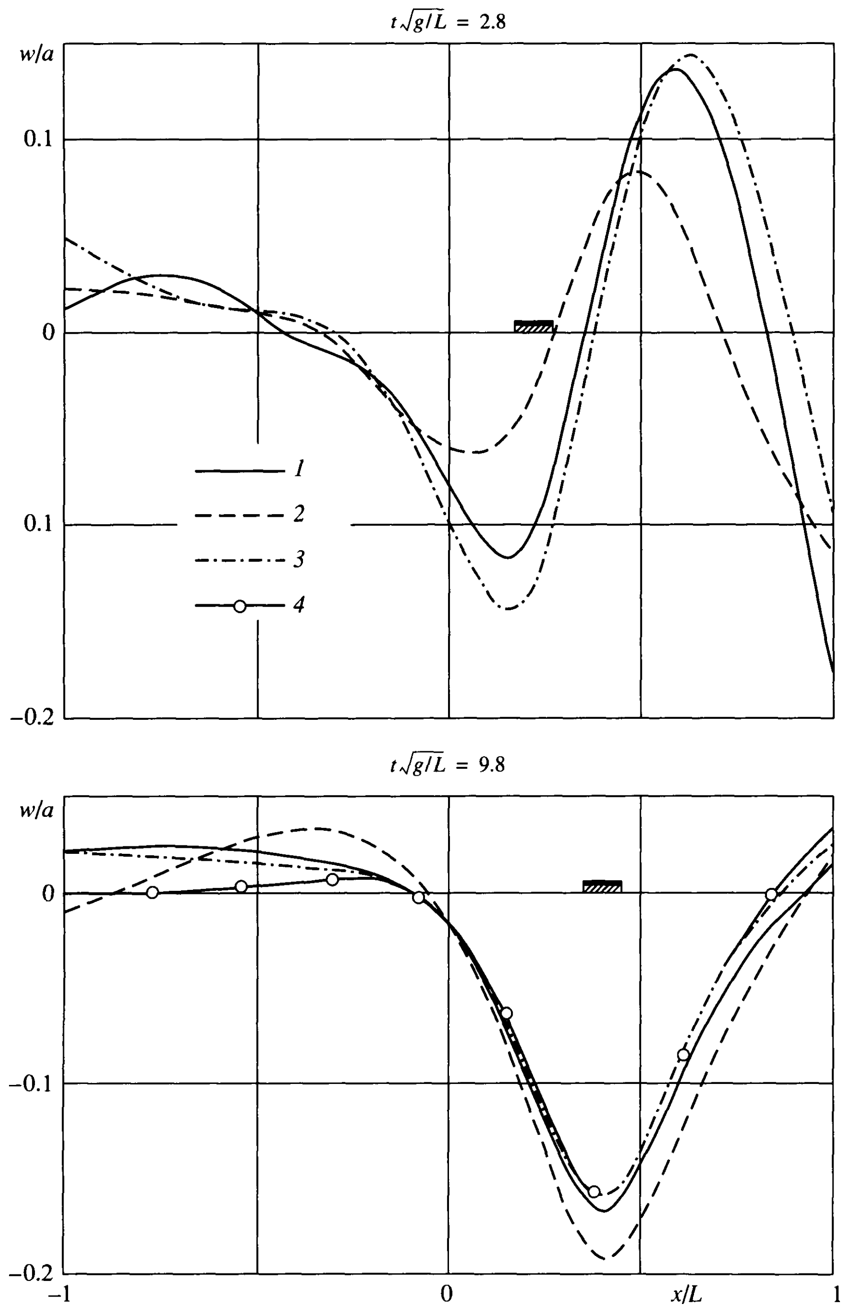

Fig. 4.

The distribution of the external load is specified in the form

$$p_e(x,t) = a\rho g\left[1 - \frac{\upsilon^2(t)}{\upsilon_0^2}e^{-\sigma t}\right]F(x,t)$$

$$F(x,t) = \begin{cases} 1 - [x - s(t)]^2/c^2 \text{ when } |x - s(t)| \le c \\ 0 \text{ when } |x - s(t)| > c \end{cases}$$

The previous initial data are used, but now

$$\delta = 5\cdot 10^{-4},\ x_0 = -0.4L,\ x_1 = 0.4L,\ c = 0.05L,\ \upsilon_0 = 0.3\sqrt{gL},\ \sigma = 3\sqrt{L/g}$$

For these values, $t_1\sqrt{g/L} \approx 5.3$.

The time dependences of the normal deflections of the beam at the instants $t\sqrt{g/L} = 2.8$ (the upper part of the figure) and $t\sqrt{g/L} = 9.8$ (the lower part) are shown in Fig. 4. The hatched rectangle shows the position of the pressure domain at a specified instant. The results for a uniform bottom, a hill and a hollow are represented by curves 1, 2, and 3. It can be seen that the oscillations of the plate are very different for the different bottom irregularities. The instant $t\sqrt{g/L} = 9.8$ corresponds to $t > t_1$. Since the motion of the load has stopped by this instant, the oscillations of the beam gradually decay and, when $t \to \infty$, they take values corresponding to a distributed static load with its centre at the point $x = x_1$. We obtain the solution of this steady-state problem from a system of differential equations which, in this case, reduces to a simple system of linear algebraic equations. The values of the static deflection are independent of the liquid depth and are shown by curve 4 in the lower part of Fig. 4. It can be seen that, in the case of a hollow and a flat bottom, the steady-state solution in the neighbourhood of the pressure domain is now fairly well satisfied. However, in the case of an irregularity in the form of a hill, the normal deflections are still far from the static value. This again shows that more persistent oscillations of the beam occur when there is a hill-type bottom irregularity.

The results obtained show that the form of the bottom irregularities can have a considerable effect on the oscillations of an elastic plate which is floating on shallow water. Bottom irregularities in the form of a hill lead to the most prolonged oscillations of the plate, other conditions being equal. The proposed method of calculation can be extended to the case of a circular plate subject to the condition that the bottom irregularity is localized and located completely under the plate. The solution of the problem of the unsteady behaviour of a circular plate floating on shallow water has been presented previously[10] in the case of a flat bottom.

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